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A CANADIAN GRID SYSTEM IN 3° TRANSVERSE MERCATOR ZONES

J. SAASTAMOINEN

Division of Applied Physics
National Research Council
Ottawa, Canada

All geodetic networks that rest previously fixed control are best computed in plane coordinates. Under this category fall a great deal of first-order triangulation and, of course, the whole volume of lower-order work—all the way down to the last monuments from which detail surveys originate.

A geodetic grid must be designed to meet first-order accuracy and should be established on federal rather than provincial level. Its use requires precomputed tables based on some conformal map projection, preferably the Transverse Mercator (Gauss-Krüger) projection. Congruent projection zones, simple scale factor, free choice of central meridian for local plane coordinates in cities and metropolitan areas—all these features of the Transverse Mercator find no parallel in any other map projection.

Following a brief introduction to the subject, a set of tables for a proposed Canadian grid system is presented.

Il est préférable de calculer en coordonnées planes tous les réseaux géodésiques qui se basent sur des contrôles déjà établis. L'on trouve dans cette catégorie une grande partie des triangulations de premier ordre et, bien entendu, toute la masse des travaux d'ordre inférieur, y compris les bornes servant de base au levé des détails.

Un réseau géodésique devrait être conçu en vue d'une précision de premier ordre et devrait être établi au niveau fédéral plutôt que provincial. Son utilisation nécéssite l'emploi de tables basées sur quelque projection cartographique conforme; de préférence, la projection Mercator Transverse (Gauss-Küger). Des zones de projections congruentes, un coéfficient d'échelle simple, le choix arbitraire du méridien central pour les coordonnées locales des villes et des régions métropolitaines, tous ces avantages de la projection Mercator ne trouvent d'équivalent dans aucune autre projection cartographique.

A la suite d'une brève introduction au sujet, on présente une série de tables pour un système de projection canadien.

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INTRODUCTION

In Canadian surveying the use of the transverse Mercator projection is almost solely limited to the national system of topographic maps. This is only natural since no survey of a local nature can be executed in any kind of a general system unless tied to geodetic control, and such control has been relatively scarce in most parts of Canada. Nor can the lack of geodetic control points be circumvented by making astronomical observations of latitude and longitude as a substitute for fundamental triangulation, because even in the most fortunate cases the accuracy of the astronomic determinations would be insufficient — of the order of ± 5 metres in ground coordinates.

Along with increasing density of geodetic control, surveyors are becoming more attracted to the use of plane coordinates, a subject frequently treated on the pages of this journal [Berry, 1963; Lilly, 1960]. The purpose of the present article is to draw attention to the great potential of the transverse Mercator projection in that it can offer a uniform, practical grid system for the whole of Canada, excepting only minor portions of the most northern Arctic islands.

As we know from the literature quoted above, the transverse Mercator plane is the conformal representation of the earth sphere or spheroid in such a way that one selected *central meridian* becomes truly rectified into a straight-line segment (*NS*, see Figure 1). It follows from the above that the equator is represented by a straight line perpendicular to the central meridian; all the other meridians and parallels are curved lines of a more complex nature. Grid coordinates are referred to the central meridian as the x-axis, and to the equator as the y-axis. The scale factor increases both eastwards and westwards from the central meridian, where it is unity, rapidly becoming excessive, and eventually infinite. Thus the effective projection area is restricted to a narrow zone embracing the central meridian. Larger areas are divided by equally

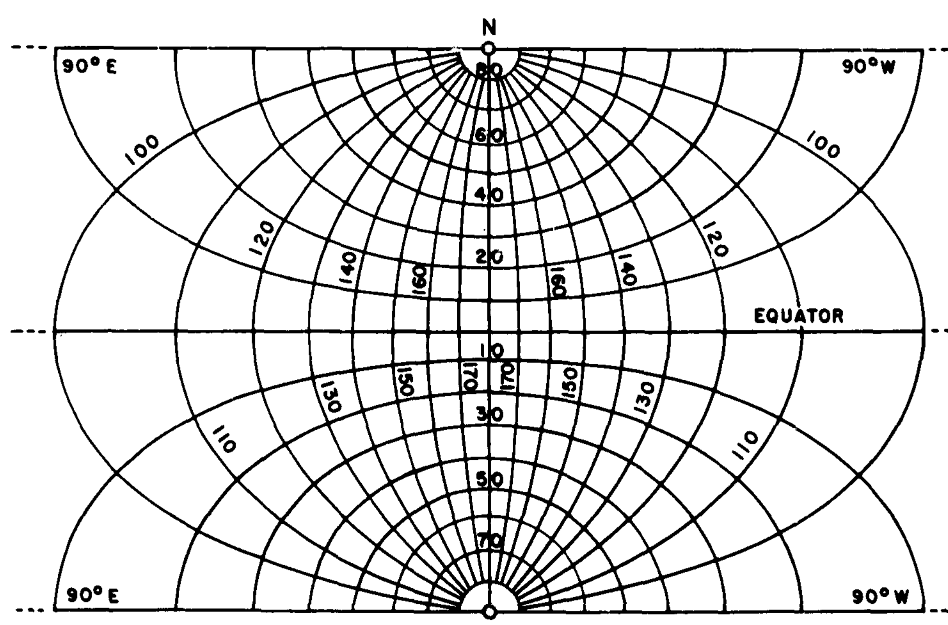

Fig. 1. Graticule of a hemisphere on the transverse Mercator projection. Note that the horizontal lines extend to infinity, and that only a narrow central zone is relatively free from exaggeration.

spaced central meridians into the number of zones required, and instead of a single grid we have a grid system.

One such grid system is that of the universal transverse Mercator (UTM) projection, the well-known military world grid in 6-degree zones, on which the effect of scale errors has been brought to a minimum by the application of a constant central scale factor of 0.9996 to all grid lengths. This means, for example, that in the vicinity of the central meridians a true length of 100 cm becomes on the projection 99.96 cm, or 0.4 mm too short. Thus maps on this projection are not fully satisfactory for the civilian user, particularly since paper shrinkage further tends to add to the above error.

A slight reduction in the zone width would be sufficient to render the scale error unimportant as far as maps are concerned. But the grid system must also serve for precise geodetic computations, which turns out to impose further conditions, also closely related to the zonal width.

Because the earth is not truly spherical (except at the poles), but a spheroid, its projection cannot be expressed by simple algebraic equations and convergent power series must be resorted to. To quote only one instance out of many, the conversion of latitude and western longitude (ϕ, λ) into grid coordinates (x, y) is effected by equations of the form:

$$x = a_0 + a_2 l^2 + a_4 l^4 + \dots$$
$$y = a_1 l + a_3 l^3 + a_5 l^5 + \dots$$

where $l = \lambda_0 - \lambda$ is the difference in longitude from central meridian λ_0, and $a_0, a_1, a_2, \dots$ is a sequence of tabulated functions of latitude ϕ. The summations must be carried on until an order of precision has been reached that is equivalent to the precision of the given geographical coordinates. It is most convenient, in view of the amount of labour involved, to keep l within about $\pm$ 2°, or the width of the zones within 4°, in which case the higher terms not indicated above can be totally omitted.

These considerations suggest the adoption of a grid system in 3-degree zones, in which each zone may be extended 30 minutes of arc beyond its ordinary longitude limits. This arrangement will provide a total overlap of 1° between adjacent zones, within which plane coordinates are expressible in both systems and provision can be made for the transformation of coordinates from one zone to the other.

TABLE I

TRANSVERSE MERCATOR 3° PROJECTION ZONES

Latitude	Zone width	Maximum scale factor	Maximum scale error
40°	256.2 km	1.000 20	1:5,000
50°	215.1 km	1.000 14	1:7,100
60°	167.4 km	1.000 09	1:11,100
70°	114.6 km	1.000 04	1:25,000
80°	58.2 km	1.000 01	1:100,000

We see from Table I that the maximum scale errors, especially those of the more important southern regions, are fairly large. However, because they have little significance for the cartographer, nothing could be gained by the introduction of a central scale factor other than unity — even though a factor of say 0.9999 would cut down the largest scale error by one-half. The maximum scale factor 1.0002 would be reduced to 1.0001; yet we find that both of these figures

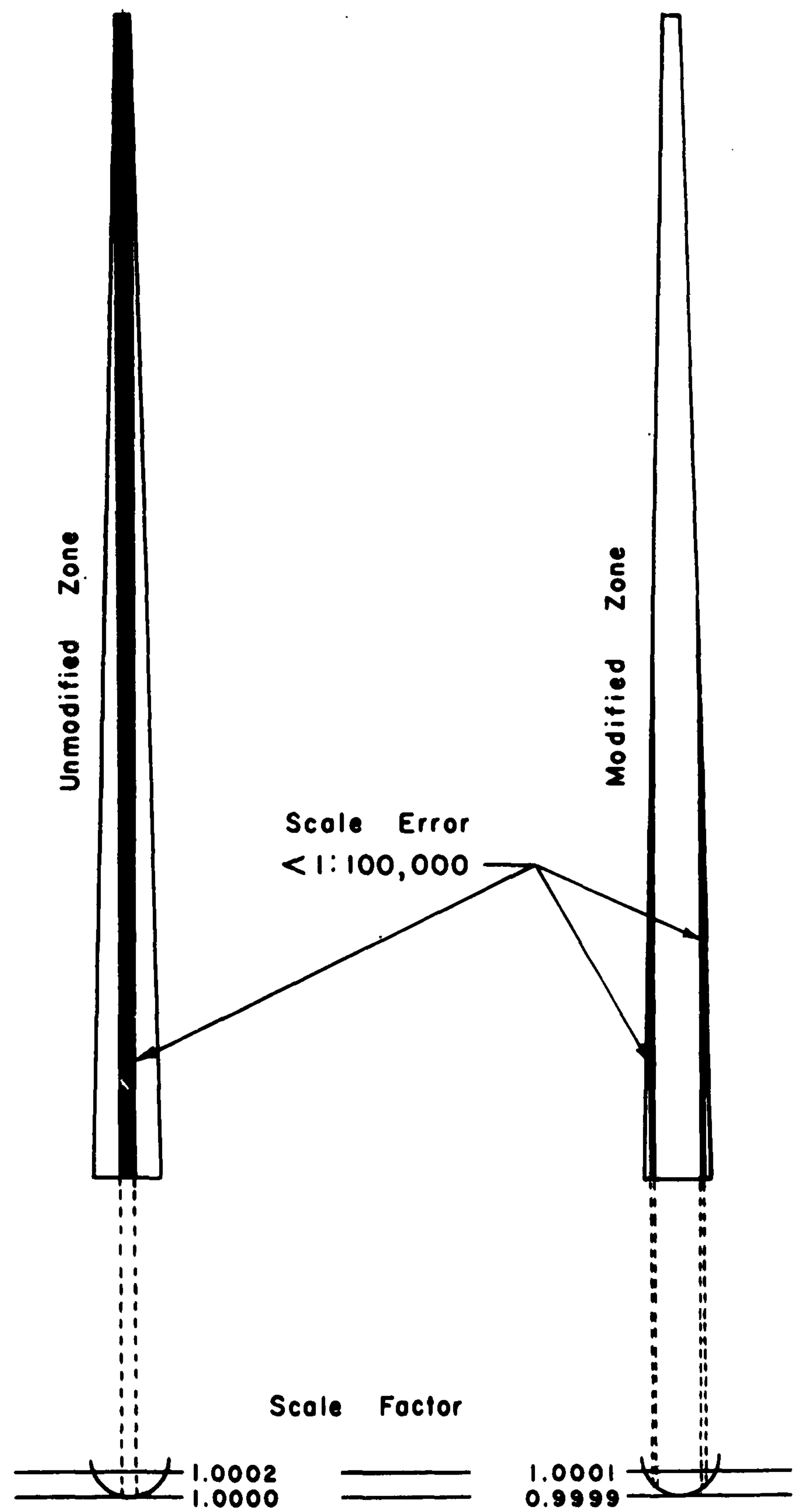

Fig. 2. Diagram showing the detrimental effect of scale modification. Black areas represent grid portions where scale correction is negligible for engineering surveys.

fall under the same order of magnitude. Each has the value of 1.000 if rounded to three decimals, and on either scale true lengths containing more than three significant numbers will usually need a scale correction, before they apply on the grid. But on the scale-modified projection similar corrections would be required in the central part of the grid as well, to the embarrassment of the surveyor. On the original projection, however, about one-third of the total grid area can be regarded as errorless in scale for most ordinary, even precise surveys (see Figure 2).

TABLE II
SCALE FACTOR

Northings x, km	$10^{17}A$
4400	1230.6
4500	1230.3
4600	1230.1
4700	1229.8
4800	1229.6
4900	1229.3
5000	1229.0
5100	1228.8
5200	1228.5
5300	1228.3
5400	1228.0
5500	1227.7
5600	1227.5
5700	1227.2
5800	1227.0
5900	1226.7
6000	1226.5
6100	1226.2
6200	1226.0
6300	1225.7
6400	1225.5
6500	1225.3
6600	1225.0
6700	1224.8
6800	1224.6
6900	1224.4
7000	1224.1
7100	1223.9
7200	1223.7
7300	1223.5
7400	1223.3
7500	1223.2
7600	1223.0
7700	1222.8
7800	1222.6
7900	1222.5
8000	1222.3
8100	1222.2
8200	1222.0
8300	1221.9
8400	1221.8
8500	1221.6
8600	1221.5
8700	1221.4
8800	1221.3
8900	1221.2

$$m = 1 + Ay^2$$

Example:

$$N = 5050{,}000$$
$$E = 75°391{,}750$$

$$10^{17}A = 1228.9$$
$$y = -108{,}250$$

$$10^{-5}y = -1.0825$$
$$10^{-10}y^2 = 1.1718$$

$$Ay^2 = 0.000\ 1440$$
$$1.000\ 0000$$
$$m = 1.000\ 1440$$

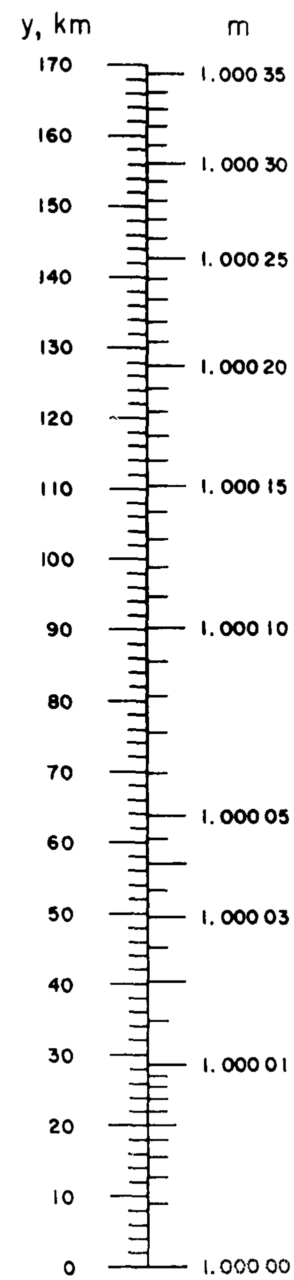

Fig. 3.
Graph for scale factor.

TABLE III
DIRECTION CORRECTION

Latitude	k	
40°	25".38	$(t_{12} - T_{12})'' = k \cdot \frac{2y_1 + y_2}{3}(x_1 - x_2)$
50	25 .32	$(t_{21} - T_{21})'' = k \cdot \frac{2y_2 + y_1}{3}(x_2 - x_1)$
60	25 .27	where x and y are expressed in units of 100 km.
70	25 .22	
80	25 .19	

Example:

Station	N	E	y
1	5050,000	75°391,750	−108,250
2	5021,650	386,000	−114,000

$x_1 - x_2 =$	+28,350	$\frac{1}{3}(2y_1 + y_2) =$	−110,170
$x_2 - x_1 =$	−28,350	$\frac{1}{3}(2y_2 + y_1) =$	−112,080

$$t_{12} - T_{12} = 25''.35(-1.1017)(+0.2835) = -7''.92$$
$$t_{21} - T_{21} = 25''.35(-1.1208)(-0.2835) = +8''.05$$

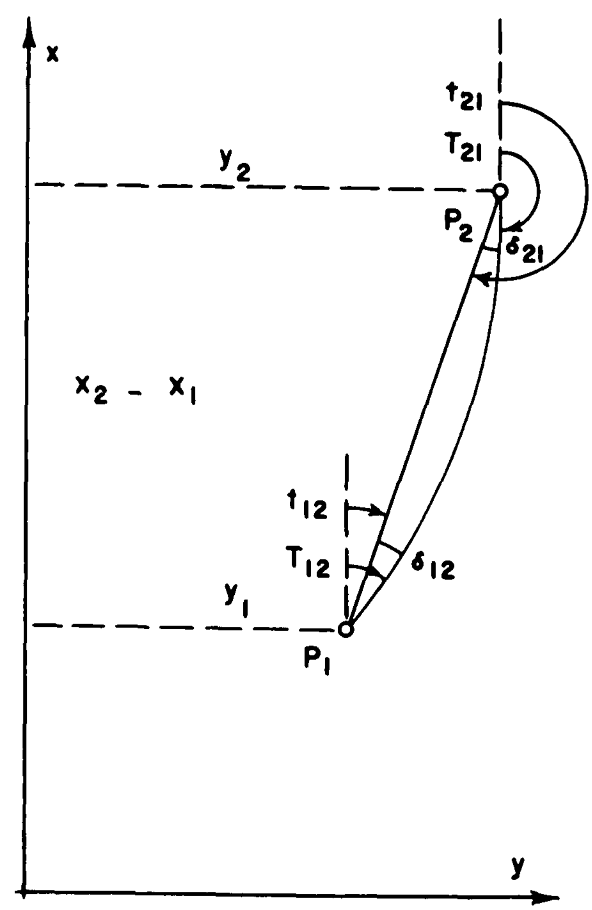

Fig. 4.

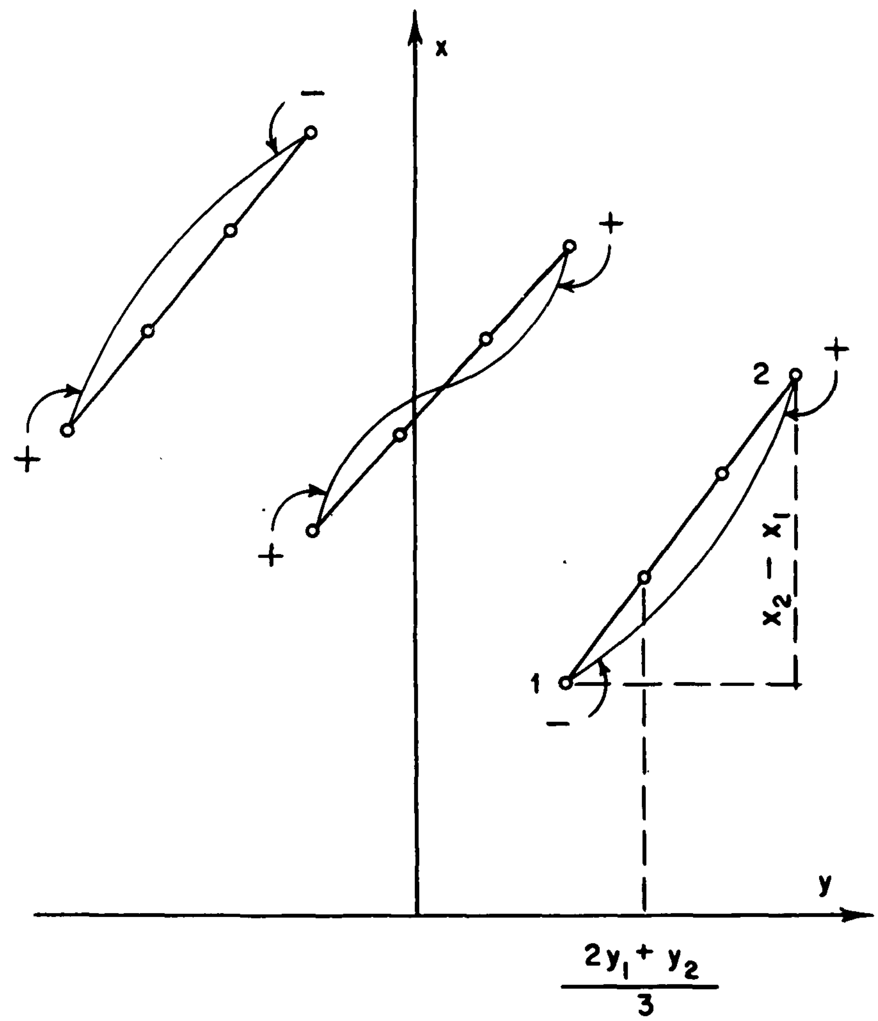

Fig. 5. Examples of algebraic sign of t-T. The chord lies between the arc and the central meridian.

A requisite for the effectiveness of plane-coordinate surveying is numerical and graphical tables, which must be made readily available for the following operations:

(a) Calculation of scale factors
(b) Calculation of projection corrections to angular observations
(c) Transformation of grid coordinates between adjacent zones
(d) Conversion of geographic positions into grid coordinates and vice versa
(e) Calculation of meridian convergences.

In addition, for his own benefit the surveyor should always make use of a *graphical grid*, on which known stations have been marked (in ink) according to their coordinates, and new points have been plotted (in pencil) according to the measured distances and angles. Drafting papers with a millimetre-grid make an excellent grid base, and the scale of the drawing should be taken large enough, depending on the type of the survey.

Tabular material contained or referred to in the latter part of this article supplies a complete set of tables for a Canadian grid system in 3-degree transverse Mercator zones. It meets the requirements for the computation of all kinds of coordinate surveys, including first-order triangulation.

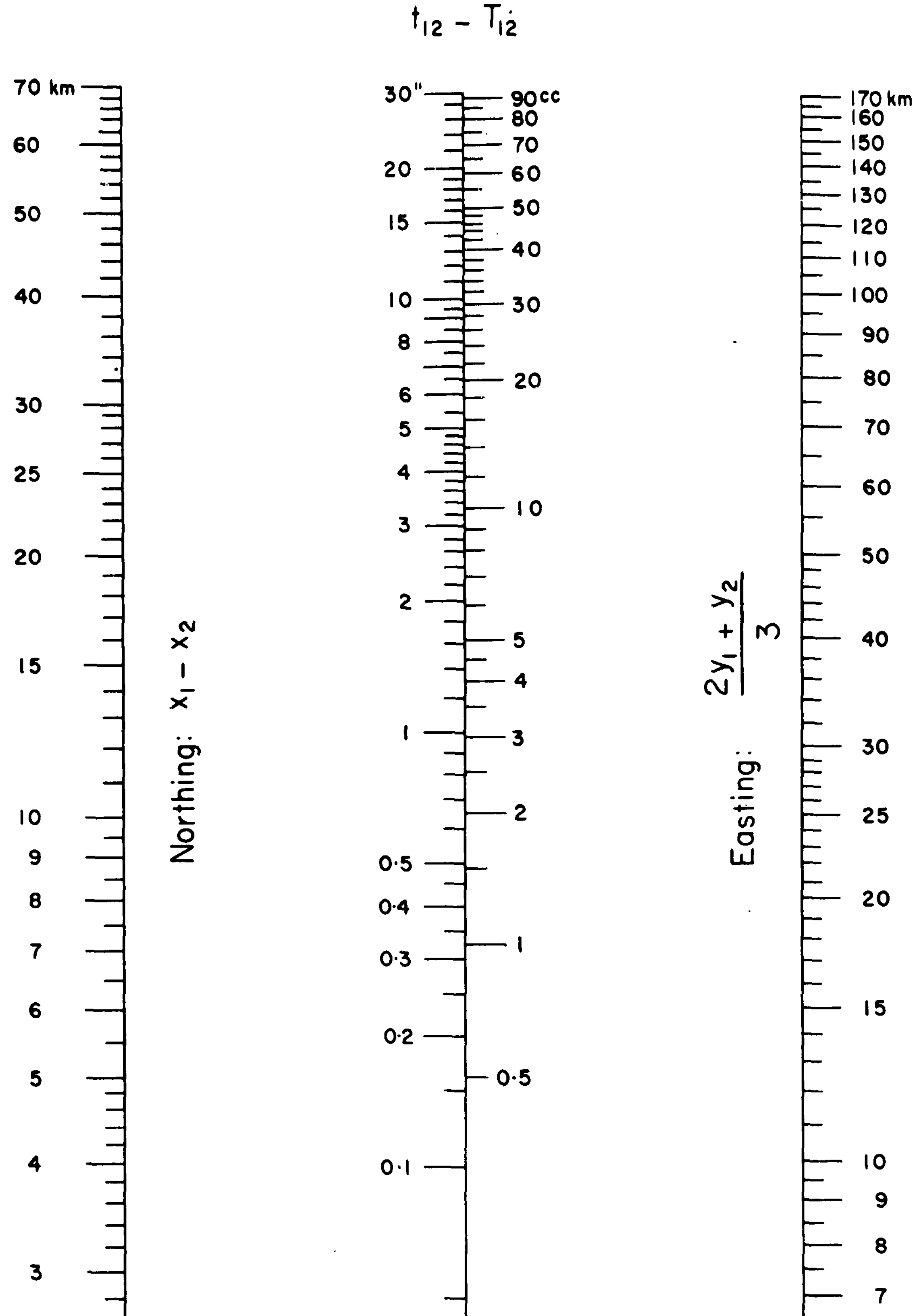

Fig. 6. Alignment chart for direction correction. Determine the coordinate values (outer scales) directly from graphical grid, and the algebraic sign of correction from Fig. 5.

EXPLANATION OF TABLES

Specifications

All the tables and nomograms (pp. 148-157) have been designed for a Canadian grid system, which has the following characteristics:

Projection:	Transverse Mercator
Reference Ellipsoid:	Clarke 1866 Spheroid
Unit of Length	Metre
Latitude Limits:	40°N to 80° N
Zone Width:	3° (4° with overlap)
Number of Zones	30
Central Meridians:	54°W, 57°W, .. 141°W
Central Scale Factor:	Unity
False Easting:	500,000

Grid reference is made in *Northings* (N) and *Eastings* (E). The central meridian can be indicated in front of the E-coordinate, as for example:

(*a*) $N =$ 6320,475.16 $E =$ 57°583,260.88 (*b*) $N =$ 4911,723.40 $E =$ 57°386,738.75

For the computation of projection corrections and for coordinate transformation, however, the above points would be notated as:

(*a*) $x =$ 6320,475.16 $y = +83,260.88$ (*b*) $x =$ 4911,723.40 $y = -113,261.25$

false easting 500,000 being subtracted from the E-coordinates.

Projection Corrections

Scale correction. — Variation of scale is of chief concern in plane-coordinate surveying and, scale corrections occur in all types of survey, except triangulation. The concept of scale correction is simple: *grid length* is the product of *true length* multiplied by *scale factor*. Difficulties may arise, however, in finding the average scale factor for a long line, such as a tellurometer measurement. The common method is to determine the tabular scale factor for the middle point of the line, although integration between the terminal coordinates y_1 and y_2 would give a more accurate value:

$$y^2 = \tfrac{1}{3}(y_1^2 + y_1y_2 + y_2^2)$$

which can be directly entered into the scale factor formula.

Regardless of length, measured distances must be corrected for slope and reduced to sea-level, to obtain the equivalent "true lengths". Methods for these reductions have been discussed elsewhere [Saastamoinen, 1962].

Direction correction. — These corrections occur in triangulation, and are not infrequently met in traversing or wherever azimuths are being tied to remote control stations.

The line of sight connecting two distant stations (P_1 and P_2, in Figure 4 on page 152) is, in general, represented on the grid by a slightly curved line with its concave side towards the central meridian. Plane computations are, however, to be carried along exactly straight lines; which implies that small corrections ($-\delta_{12}$, $+\delta_{21}$) must be applied to the observed directions of the line of sight. Clear distinction has to be made between two kinds of grid azimuths, viz. the *plane azimuth* (t_{12}) of the chord P_1P_2, and the *projected grid azimuth* (T_{12}) of the arc P_1P_2.

Differences $t-T$ are called *direction corrections*. In the measurement of horizontal angles, each line of sight has its individual direction correction, which must be determined and added algebraically to the observed value of the direction.

TABLE IV

ZONAL TRANSFORMATION OF GRID COORDINATES

$$(x_1, y_1) \rightarrow (x_2, y_2)$$

$$\begin{cases} X = x_1 - x_0 \\ Y = y_1 - y_0 \end{cases} \qquad \begin{cases} X' = X^2 - Y^2 \\ Y' = 2XY \end{cases} \qquad \begin{cases} X'' = X(X^2 - 3Y^2) \\ Y'' = Y(3X^2 - Y^2) \end{cases}$$

$$\begin{cases} x_2 = x_1 - AY - BX - CY' - DX' + EY'' + FX'' \\ y_2 = -y_0 + Y + AX - BY + CX' - DY' - EX'' + FY'' \end{cases}$$

Sign Convention: Initial coordinate y_1 is always taken as positive. (In the transformation of eastern coordinates to the western zone, change the algebraic sign of y_1 and of y_2.)

Example:

$N =$	5018,505.68	$E =$	78°659,359.03	X^2	0.052368×10^{10}
				XY	-0.096385
				Y^2	0.177398
x_1	5018,505.680	y_1	159,359.030		
x_0	5041,389.745	y_0	117,240.379	X'	-0.125030×10^{10}
X	−22,884.065	Y	+42,118.651	Y'	-0.192770
		$-y_0 + Y$	−75,121.728		
A	0.0373 4125	C	28.76×10^{-10}	E	$0.16 \quad \times 10^{-15}$
B	0.0006 9743	D	1.61	F	–
$-AY$	−1,572.763	$+AX$	−854.520	$X^2 - 3Y^2$	$-0.4798 \quad \times 10^{10}$
$-BX$	+15.960	$-BY$	−29.375	$3X^2 - Y^2$	-0.0203
$-CY'$	+5.544	$+CX'$	−3.596		
$-DX'$	+0.201	$-DY'$	+0.310	X''	$+0.1098 \quad \times 10^{15}$
$+EY''$	−0.001	$-EX''$	−0.018	Y''	-0.0086
$+FX''$	0	$+FY''$	0		
Sum	−1,551.059		−887.199		
x_2	5016,954.621	y_2	−76,008.927		
$N =$	5016,954.62	$E =$	75°423,991.07		

ϕ_0	x_0	y_0	A	B	$10^{10} \cdot C$	$10^{10} \cdot D$	$10^{15} \cdot E$	$10^{15} \cdot F$
40° 0′	4430 396.857	128 096.723	0.0336 5449	0.0005 6647	31.48	1.59	0.14	
30	4485 918.802	127 157.329	0.0340 0301	0.0005 7827	31.24	1.60	0.14	
41 0	4541 445.289	126 208.209	0.0343 4893	0.0005 9010	31.00	1.60	0.14	
30	4596 976.334	125 249.435	0.0346 9224	0.0006 0196	30.76	1.60	0.14	
42 0	4652 511.945	124 281.078	0.0350 3290	0.0006 1384	30.52	1.61	0.15	
30	4708 052.132	123 303.209	0.0353 7088	0.0006 2575	30.28	1.61	0.15	
43 0	4763 596.905	122 315.901	0.0357 0617	0.0006 3767	30.03	1.61	0.15	
30	4819 146.270	121 319.228	0.0360 3874	0.0006 4961	29.78	1.61	0.15	
44 0	4874 700.234	120 313.263	0.0363 6855	0.0006 6156	29.53	1.61	0.15	
30	4930 258.798	119 298.083	0.0366 9559	0.0006 7351	29.28	1.61	0.15	
45 0	4985 821.968	118 273.763	0.0370 1984	0.0006 8547	29.02	1.61	0.15	
30	5041 389.745	117 240.379	0.0373 4125	0.0006 9743	28.76	1.61	0.16	
46 0	5096 962.128	116 198.009	0.0376 5983	0.0007 0938	28.50	1.61	0.16	
30	5152 539.115	115 146.730	0.0379 7553	0.0007 2133	28.24	1.61	0.16	
47 0	5208 120.705	114 086.620	0.0382 8834	0.0007 3327	27.98	1.61	0.16	
30	5263 706.893	113 017.760	0.0385 9822	0.0007 4519	27.71	1.61	0.16	
48 0	5319 297.672	111 940.230	0.0389 0517	0.0007 5709	27.44	1.60	0.16	
30	5374 893.037	110 854.111	0.0392 0915	0.0007 6897	27.17	1.60	0.16	0.01
49 0	5430 492.978	109 759.483	0.0395 1014	0.0007 8083	26.90	1.60	0.16	0.01
30	5486 097.486	108 656.429	0.0398 0811	0.0007 9266	26.63	1.59	0.16	0.01
50 0	5541 706.551	107 545.032	0.0401 0305	0.0008 0446	26.35	1.59	0.17	0.01

ϕ_0	x_0	y_0	A	B	$10^{10}\cdot C$	$10^{10}\cdot D$	$10^{15}\cdot E$	$10^{16}\cdot F$
50 0	5541 706.551	107 545.032	0.0401 0305	0.0008 0446	26.35	1.59	0.17	0.01
30	5597 320.158	106 425.375	0.0403 9494	0.0008 1621	26.07	1.58	0.17	0.01
51 0	5652 938.294	105 297.543	0.0406 8375	0.0008 2793	25.79	1.58	0.17	0.01
30	5708 560.944	104 161.620	0.0409 6946	0.0008 3960	25.51	1.57	0.17	0.01
52 0	5764 188.092	103 017.692	0.0412 5204	0.0008 5123	25.23	1.56	0.17	0.01
30	5819 819.717	101 865.846	0.0415 3148	0.0008 6280	24.94	1.56	0.17	0.01
53 0	5875 455.803	100 706.168	0.0418 0775	0.0008 7433	24.65	1.55	0.17	0.01
30	5931 096.326	99 538.745	0.0420 8085	0.0008 8579	24.36	1.54	0.17	0.01
54 0	5986 741.266	98 363.667	0.0423 5073	0.0008 9719	24.07	1.53	0.17	0.01
30	6042 390.599	97 181.021	0.0426 1738	0.0009 0853	23.78	1.52	0.18	0.01
55 0	6098 044.299	95 990.897	0.0428 8079	0.0009 1980	23.49	1.51	0.18	0.01
30	6153 702.340	94 793.385	0.0431 4094	0.0009 3100	23.19	1.50	0.18	0.01
56 0	6209 364.695	93 588.575	0.0433 9779	0.0009 4213	22.89	1.49	0.18	0.01
30	6265 031.335	92 376.560	0.0436 5134	0.0009 5317	22.59	1.48	0.18	0.01
57 0	6320 702.229	91 157.431	0.0439 0157	0.0009 6414	22.29	1.47	0.18	0.01
30	6376 377.347	89 931.280	0.0441 4845	0.0009 7502	21.99	1.46	0.18	0.01
58 0	6432 056.654	88 698.201	0.0443 9197	0.0009 8581	21.68	1.45	0.18	0.01
30	6487 740.117	87 458.286	0.0446 3211	0.0009 9651	21.38	1.43	0.18	0.01
59 0	6543 427.701	86 211.631	0.0448 6884	0.0010 0711	21.07	1.42	0.18	0.01
30	6599 119.368	84 958.330	0.0451 0216	0.0010 1762	20.76	1.41	0.19	0.01
60 0	6654 815.081	83 698.478	0.0453 3205	0.0010 2803	20.45	1.39	0.19	0.01
30	6710 514.800	82 432.172	0.0455 5848	0.0010 3833	20.14	1.38	0.19	0.01
61 0	6766 218.485	81 159.507	0.0457 8144	0.0010 4852	19.82	1.36	0.19	0.01
30	6821 926.095	79 880.581	0.0460 0092	0.0010 5860	19.51	1.35	0.19	0.01
62 0	6877 637.586	78 595.491	0.0462 1690	0.0010 6857	19.19	1.33	0.19	0.01
30	6933 352.915	77 304.336	0.0464 2935	0.0010 7842	18.88	1.32	0.19	0.01
63 0	6989 072.036	76 007.214	0.0466 3827	0.0010 8816	18.56	1.30	0.19	0.01
30	7044 794.901	74 704.223	0.0468 4365	0.0010 9777	18.24	1.28	0.19	0.01
64 0	7100 521.466	73 395.465	0.0470 4545	0.0011 0725	17.92	1.27	0.19	0.01
30	7156 251.679	72 081.037	0.0472 4367	0.0011 1661	17.59	1.25	0.19	0.02
65 0	7211 985.492	70 761.042	0.0474 3830	0.0011 2583	17.27	1.23	0.19	0.02
30	7267 722.853	69 435.580	0.0476 2931	0.0011 3492	16.94	1.21	0.20	0.02
66 0	7323 463.709	68 104.752	0.0478 1670	0.0011 4387	16.62	1.19	0.20	0.02
30	7379 208.009	66 768.662	0.0480 0045	0.0011 5269	16.29	1.18	0.20	0.02
67 0	7434 955.699	65 427.410	0.0481 8055	0.0011 6136	15.96	1.16	0.20	0.02
30	7490 706.721	64 081.101	0.0483 5697	0.0011 6988	15.63	1.14	0.20	0.02
68 0	7546 461.020	62 729.837	0.0485 2973	0.0011 7826	15.30	1.12	0.20	0.02
30	7602 218.541	61 373.721	0.0486 9878	0.0011 8649	14.97	1.10	0.20	0.02
69 0	7657 979.222	60 012.859	0.0488 6413	0.0011 9457	14.63	1.07	0.20	0.02
30	7713 743.004	58 647.354	0.0490 2576	0.0012 0249	14.30	1.05	0.20	0.02
70° 0′	7769 509.829	57 277.312	0.0491 8366	0.0012 1025	13.96	1.03	0.20	0.02
30	7825 279.634	55 902.839	0.0493 3781	0.0012 1785	13.63	1.01	0.20	0.02
71 0	7881 052.358	54 524.038	0.0494 8822	0.0012 2529	13.29	0.99	0.20	0.02
30	7936 827.938	53 141.018	0.0496 3486	0.0012 3257	12.95	0.97	0.20	0.02
72 0	7992 606.308	51 753.884	0.0497 7771	0.0012 3968	12.61	0.94	0.20	0.02
30	8048 387.407	50 362.744	0.0499 1679	0.0012 4662	12.27	0.92	0.20	0.02
73 0	8104 171.164	48 967.704	0.0500 5206	0.0012 5339	11.93	0.90	0.20	0.02
30	8159 957.517	47 568.871	0.0501 8352	0.0012 5999	11.59	0.87	0.20	0.02
74 0	8215 746.396	46 166.356	0.0503 1117	0.0012 6641	11.25	0.85	0.21	0.02
30	8271 537.734	44 760.263	0.0504 3499	0.0012 7265	10.90	0.83	0.21	0.02
75 0	8327 331.464	43 350.703	0.0505 5497	0.0012 7872	10.56	0.80	0.21	0.02
30	8383 127.513	41 937.785	0.0506 7111	0.0012 8461	10.21	0.78	0.21	0.02
76 0	8438 925.812	40 521.617	0.0507 8339	0.0012 9031	9.87	0.75	0.21	0.02
30	8494 726.290	39 102.309	0.0508 9181	0.0012 9583	9.52	0.73	0.21	0.02
77 0	8550 528.876	37 679.971	0.0509 9636	0.0013 0116	9.18	0.70	0.21	0.02
30	8606 333.497	36 254.712	0.0510 9703	0.0013 0631	8.83	0.68	0.21	0.02
78 0	8662 140.079	34 826.643	0.0511 9381	0.0013 1126	8.48	0.65	0.21	0.02
30	8717 948.552	33 395.874	0.0512 8669	0.0013 1603	8.13	0.63	0.21	0.02
79 0	8773 758.837	31 962.516	0.0513 7568	0.0013 2060	7.78	0.60	0.21	0.02
30	8829 570.862	30 526.680	0.0514 6076	0.0013 2498	7.43	0.58	0.21	0.02
80 0	8885 384.551	29 088.477	0.0515 4192	0.0013 2917	7.08	0.55	0.21	0.02

Zonal Transformations

Table IV for zonal transformations is based upon the method due to Hirvonen [1938], whose original formulae for the tabular functions have been adapted for Canadian latitudes. Interpolation is not required in the use of the table. An error rarely exceeding 2 or 3 millimetres may be expected in the transformed coordinates because of accumulation of rounding errors in the computation.

Transformations Between Spheroid and Grid

Special tables for the conversion between geographic positions and grid coordinates have not been prepared at the present time. The existing tables for the universal transverse Mercator projection [Army Map Service, 1948] can be used for this purpose, as well as for the calculation of meridian convergences, by taking the UTM central scale factor into account in the computation. Thus, when geographic positions are being converted into grid coordinates the resulting x and y values must be divided by 0.9996. Conversely, grid coordinates x and y must be multiplied by 0.9996 before UTM tables are used for the determination of geographic positions.

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